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ON INTUITIONISTIC TRIANGULAR FUZZY NUMBERS

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Abstract: Intuitionistic fuzzy set was introduced by Atanassov (1986) as a generalization of fuzzy set. The notions of Triangular fuzzy numbers and fuzzy arithmetic are vital parts of fuzzy set Theory. The paper is therefore addressed to the introduction of intuitionistic Triangular fuzzy numbers where proposed division is based on (α, β) cut process.

Keywords and Phrases: Intuitionistic fuzzy set, Fuzzy numbers, Intuitionistic Triangular fuzzy numbers, (α, β) cut.

2010 Mathematics Subject Classification: 94D05, 03E72.

1. Introduction.

Fuzzy set Theory has been introduced by Lofti A. Zadeh [1] which permits the gradual assessments of the membership of elements in a set described in the interval [0,1]. As a generalization of the concept of fuzzy sets, Atanassov [4] introduced Intuitionistic fuzzy sets by taking membership and non-membership grades for the same element of the universal set of discourse. Fuzzy numbers and Triangular fuzzy numbers have their importance in fuzzy set theory [2], [3]. We in this paper have developed the concept of Intuitionistic Triangular fuzzy numbers and have shown how arithmetic operations specially proposed division is based on (α, β) cut method. Figures have been sketched and some numerical have been placed for justification.

2. Preliminaries

A fuzzy set $\tilde{A}$ on a non empty set X is characterised by its membership function $\mu_{\tilde{A}} : X \to [0, 1]$ where $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of elements

of X in fuzzy set $\tilde{A}$ for each $x \in X$ [1].
A fuzzy set $\tilde{A}$ is called normal if $\mu_{\tilde{A}}(x) = 1$ for at least are $x \in X$ and the $\alpha-$ Level set or $\alpha-$ cut of a fuzzy set $\tilde{A}$ on X is a crisp set defined by

$$\tilde{A}_\alpha = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\}, \quad x \in [0, 1]$$

Definition 2.1. Fuzzy numbers are close to a given real numbers or around a given interval of real numbers. To qualify as a fuzzy real number, a fuzzy set $\tilde{A}$ on R must be
(i) A normal fuzzy set
(ii) The α- cut A_α must be a closed interval for every $\alpha \in [0, 1]$
(iii) The support set $O + \tilde{A}$ must be bounded.
Each fuzzy number is uniquely determined by its α-cut. So the arithmetic operations mean the arithmetic operations on closed intervals.
We define

$$[a, b] + [d, e] = [a + d, b + e]$$

$$[a, b] - [d, e] = [a - e, b - d]$$

$$[a, b].[d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]$$

$$[a, b] \div [d, e] = [a, b].\left[\frac{1}{d}, \frac{1}{e}\right] \quad \text{provided} \quad 0 \neq [d, e][2]$$

Definition 2.2. A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & x > a_3 \quad [3] \end{cases}$$

Example.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{for} \quad x \leq -1 \quad \text{and} \quad x > 3 \\ \dfrac{x + 1}{2}, & \text{for} \quad -1 < x \leq 1 \\ \dfrac{3 - x}{2}, & \text{for} \quad 1 < x \leq 3 \end{cases}$$

is a triangular fuzzy number, figure (a) $(-1, 1, 3)$

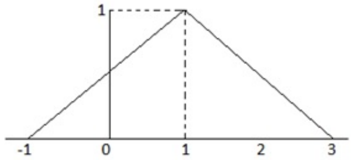

Fig. (a)

Definition 2.3. An intuitionistic fuzzy set (IFS) A on X is defined as the set of all ordered triplets of the form

$$A = \{(x, \mu_A(x), \mu_A^1(x)) : x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ and $\mu_A^1 : X \to [0, 1]$ define the degree of membership and degree of non-membership of the elements $x \in X$ respectively and for every $x \in X$ in A, $0 \leq \mu_A(x) + \mu_A^1(x) \leq 1$ [4].

Example. Let $X = \{a, b, c, d\}$, then $A = \{(a, .1, .9), (b, 0, .9), (c, .2, .9), (d, .3, .6)\}$ is an intuitionistic fuzzy set. The support of an intuitionistic fuzzy set A on R is the crisp set of all $x \in R$ with

$$\mu_A(x) + \mu_A^1(x) \leq 1; \mu_A^1(x) > 0, \mu_A(x) > 0$$

In the above example supp.$(A) = \{a, c, d\}$.

Definition 2.4. The (α, β) cut or (α, β) level interval of an intuitionistic fuzzy set A, where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$ is a crisp set of elements of X denoted by $A_{\alpha,\beta} = \{x \in X / \mu_A(x) < \alpha \text{ and } \mu_A^1(x) \leq \beta\}$ which belong to A at least to the degree α and which does belong to A at least to the degree β [5].

Example. Let $X = \{a, b, c, d\}$ and $A = \{(a, .1, .9), (b, 0, .9), (c, .7, .2), (d, .7, .2)\}$ be an intuitionistic fuzzy set. Take $\alpha = .2$ and $\beta = .7$
Clearly $\alpha, \beta \in [0, 1]$ with $\alpha + \beta < 1$.
Then $A_{\alpha,\beta} = \{d\}$

3. Intuitionistic Triangular Fuzzy Number

An intuitionistic Triangular fuzzy number (ITFN) is an intuitionistic fuzzy number in R with the following membership function $\mu_A(x)$ and of non-membership function $\mu_A^1(x)$.

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for} \quad a_1 \leq x \leq a_2 \\ \dfrac{x - a_3}{a_2 - a_3}, & \text{for} \quad a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_A^1(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1}, & \text{for} \quad a_1 \le x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2}, & \text{for} \quad a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases}$$

Where $a_1^1 \le a_1 \le a_2 \le a_3 \le a_3^1$ and $\mu_A(x) + \mu_A^1(x) \le 1$ or $\mu_A(x) = \mu_A^1(x)$ for all $x \in R$. This intuitionistic Triangular fuzzy number is denoted by $A = (a_1, a_2, a_3 : a_1^1, a_2, a_3^1)$ consisting of six triple number, figure (b).
or $A = \{(a_1, a_2, a_3); (a_1^1, a_2, a_3^1)\}$

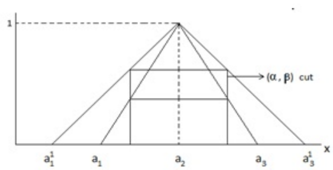

Fig. (b)

4. Arithmetic Operations

The arithmetic operations $+, -, \times$ of intuitionistic Triangular fuzzy number are defined as follows:
Let $A = \{(a_1, a_2, a_3); (a_1^1, a_2, a_3^1)\}$ and $B = \{(b_1, b_2, b_3); (b_1^1, b_2, b_3^1)\}$ be two intuitionistic Triangular fuzzy numbers, Then

(i) $A + B = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a_1^1 + b_1^1, a_2 + b_2, a_3^1 + b_3^1)\}$

(ii) $A - B = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1); (a_1^1 - b_3^1, a_2 - b_2, a_3^1 - b_1^1)\}$

(iii) $A \times B = \{(a_1 b_1, a_2 b_2, a_3 b_3); (a_1^1 b_1^1, a_2 b_2, a_3^1 b_3^1)\}$ are also intuitionistic Triangular fuzzy numbers.

Justification (4.1):
Let $A = \{(1, 2, 3); (.5, 2, 3.5)\}$ and $B = \{(4, 5, 6); (3.5, 5, 6.5)\}$ be two intuitionistic Triangular fuzzy numbers given by figure (c).

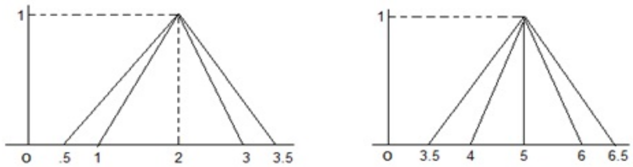

Fig. (c)

Then their sum, difference and product are obtained by arithmetic operations as follows

(i) $A + B = \{(1, 2, 3); (.5, 2, 3.5)\} + \{(4, 5, 6); (3.5, 5, 6.5)\}$
$= \{(1 + 4, 2 + 5, 3 + 6); (.5 + 3.5, 2 + 5, 3.5 + 6.5)\}$
$= \{(5, 7, 9); (4, 7, 10)\}$ figure (d).

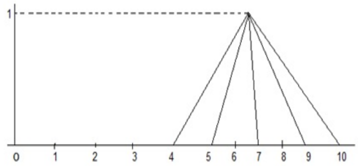

Fig. (d)

(ii) $A - B = \{(1, 2, 3); (.5, 2, 3.5)\} - \{(4, 5, 6); (3.5, 5, 6.5)\}$
$= \{(1 - 6, 2 - 5, 3 - 4); (.5 - 6.5, 2 - 5, 3.5 - 3.5)\}$
$= \{(-5, -3, -1); (-6, -3, 0)\}$ figure (e).

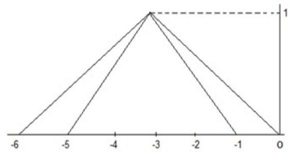

Fig. (e)

(iii) $A \times B = \{(1, 2, 3); (.5, 2, 3.5)\} \times \{(4, 5, 6); (3.5, 5, 6.5)\}$
$= \{(1 \times 4, 2 \times 5, 3 \times 6); (.5 \times 3.5, 2 \times 5, 3.5 \times 6.5)\}$
$= \{(4, 10, 18); (1.7, 10, 22.7)\}$ figure (f).

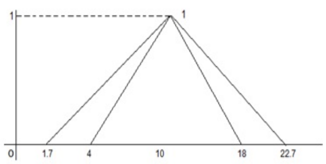

Fig. (f)

5. Division of two Intuitionistic Triangular Fuzzy Numbers

Let $A = \{(a_1, a_2, a_3); (a_1^1, a_2, a_3^1)\}$ and $B = \{(b_1, b_2, b_3); (b_1^1, b_2, b_3^1)\}$ be two intuitionistic Triangular fuzzy numbers, Then $\dfrac{A}{B} = \left\{\left(\dfrac{a_1}{b_3}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_1}\right); \left(\dfrac{a_1^1}{b_3^1}, \dfrac{a_2}{b_2}, \dfrac{a_3^1}{b_1^1}\right)\right\}$ is also intuitionistic Triangular fuzzy number.

Proof. Let $u = \dfrac{x}{y}$ be the Transformation with the membership function and non-membership function of intuitionistic Triangular fuzzy numbers. Then $\tilde{u} = \dfrac{\tilde{A}^1}{\tilde{B}^1}$ can be found by (α, β)- cut method.

Remark

(i) α-cut for membership function of $\tilde{A}^1$ is $[a_1+\alpha(a_2-a_1), a_3-\alpha(a_3-a_2)]$ $\forall\alpha \in [0, 1]$.
(ii) α-cut for membership function of $\tilde{B}^1$ is $[b_1+\alpha(b_2-b_1), b_3-\alpha(b_3-b_2)]$ $\forall\alpha \in [0, 1]$.

To calculate the division of intuitionistic Triangular fuzzy numbers $\tilde{A}^1$ and $\tilde{B}^1$, we first divide the α-cut of $\tilde{A}^1$ and $\tilde{B}^1$ using interval arithmetic

$$\frac{\tilde{A}^1}{\tilde{B}^1} = \frac{a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)}{b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)}$$

$$= \left[\frac{a_1 + \alpha(a_2 - a_1)}{b_3 - \alpha(b_3 - b_2)}, \frac{a_3 - \alpha(a_3 - a_2)}{b_1 + \alpha(b_2 - b_1)}\right]$$

Equating to x both the components

$$x = \frac{a_1 + \alpha(a_2 - a_1)}{b_3 - \alpha(b_3 - b_2)}, \qquad x = \frac{a_3 - \alpha(a_3 - a_2)}{b_1 + \alpha(b_2 - b_1)}$$

$$\Rightarrow \quad \alpha = \frac{b_3x - a_1}{(a_2 - a_1) + (b_3 - b_2)x} \quad \text{and} \quad \alpha = \frac{a_3 - b_1x}{(a_3 - a_2) + (b_2 - b_1)}$$

when $\alpha = 0$, $b_3x - a_1 = 0 \Rightarrow x = \dfrac{a_1}{b_3}$ and $a_3 - b_1x = 0 \Rightarrow x = \dfrac{a_3}{b_1}$ so setting $\alpha = 0$ and $\alpha = 1$, we have

$$\mu_{\frac{\tilde{A}^1}{\tilde{B}^1}}(x) = \begin{cases} \dfrac{b_3x - a_1}{(a_2 - a_1) + (b_3 - b_2)x}, & \dfrac{a_1}{b_3} \leq x \leq \dfrac{a_2}{b_2} \\[2ex] \dfrac{a_3 - b_1x}{(a_3 - a_2) + (b_2 - b_1)x}, & \dfrac{a_2}{b_2} \leq x \leq \dfrac{a_3}{b_1} \end{cases}$$

(iii) β-cut for non-membership function of $\tilde{A}^1$ is $[a_2-\beta(a_2-a_1^1), a_2+\beta(a_3^1-a_2)]$ $\forall\beta \in [0, 1]$.

(iv) β-cut for non-membership function of $\tilde{B}^1$ is $[b_2 - \beta(b_2 - b_1^1), b_2 + \beta(b_3^1 - b_2)]$. To evaluate the division of intuitionistic Triangular fuzzy numbers $\tilde{A}^1$ and $\tilde{B}^1$, we first divide the β-cut of $\tilde{A}^1$ and $\tilde{B}^1$ using interval arithmetic

$$\frac{\tilde{A}^1}{\tilde{B}^1} = \frac{a_2 - \beta(a_2 - a_1^1), a_2 + \beta(a_3^1 - a_2)}{b_2 - \beta(b_2 - b_1^1), b_2 + \beta(b_3^1 - b_2)}$$

$$= \left[\frac{a_2 - \beta(a_2 - a_1^1)}{b_2 + \beta(b_3^1 - b_2)}, \frac{a_2 + \beta(a_3^1 - a_2)}{b_2 - \beta(b_2 - b_1^1)}\right]$$

Equating to x both the components

$$x = \frac{a_2 - \beta(a_2 - a_1^1)}{b_2 + \beta(b_3^1 - b_2)}, \qquad x = \frac{a_2 + \beta(a_3^1 - a_2)}{b_2 - \beta(b_2 - b_1^1)}$$

Expressing β in terms of x and setting $\beta = 0, \beta = 1$, we have

$$\mu_{\frac{\tilde{A}^1}{\tilde{B}^1}}(x) = \begin{cases} \dfrac{a_2 - b_2 x}{(a_2 - a_1^1) + (b_3^1 - b_2)x}, & \dfrac{a_1^1}{b_3^1} \le x \le \dfrac{a_2}{b_2} \\[2ex] \dfrac{b_2 x - a_2}{(a_3^1 - a_2) + (b_2 - b_1^1)x}, & \dfrac{a_2}{b_2} \le x \le \dfrac{a_3}{b_1^1} \end{cases}$$

Hence, Division rule is proved for membership and non-membership function.
$\dfrac{A}{B} = \left\{\left(\dfrac{a_1}{b_3}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_1}\right); \left(\dfrac{a_1^1}{b_3^1}, \dfrac{a_2}{b_2}, \dfrac{a_3^1}{b_1^1}\right)\right\}$ is also intuitionistic Triangular fuzzy number.
Justification (5.2): Let $A = \{(1, 2, 3); (.5, 2, 3.5)\}$ and $\{(4, 5, 6); (3.5, 5, 6.5)\}$ be two intuitionistic Triangular fuzzy numbers.
Then assuming them to be $\{(a_1, a_2, a_3); (a_1^1, a_2, a_3^1)\}$ and $\{(b_1, b_2, b_3); (b_1^1, b_2, b_3^1)\}$

$$\frac{A}{B} = \left\{\left(\frac{1}{6}, \frac{2}{5}, \frac{3}{4}\right); \left(\frac{.5}{6.5}, \frac{2}{5}, 1\right)\right\}$$

$$= \{(.17, .4, .8); (.07, .4, 1)\}, \qquad \text{figure(g)}$$

which is an intuitionistic Triangular fuzzy numbers.

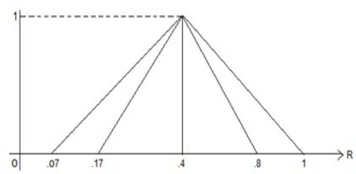

Fig. (g)

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